

3.6 Section Algebraic Approach

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Example: $f'(x)$ when $f(x) = x^2 + 3x - 2$

Step 1 $f(x+h) = (x+h)^2 + 3(x+h) - 2$

Alternate way to look @ it: $f(\quad) = (\quad)^2 + 3(\quad) - 2$

Step 2 $f(x+h) = (x+h)^2 + 3(x+h) - 2$
 $= (x+h)(x+h) + 3(x+h) - 2$

$$= x^2 + 2xh + h^2 + 3x + 3h - 2$$

Step 3 $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

Plug in:

$$\lim_{h \rightarrow 0} \frac{(x^2 + 2xh + h^2 + 3x + 3h - 2) - (\cancel{x^2} + \cancel{3x} - 2)}{h}$$

Step 4 $\lim_{h \rightarrow 0} \frac{2xh + h^2 + 3h}{h}$

Step 5 $\lim_{h \rightarrow 0} 2x + h + 3$

Step 6 Plug in $h=0$; $2x+3$

Example: $f(x) = \frac{3}{x-2}$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Step 1 Find $f(x+h) = \frac{3}{(x+h)-2}$

Step 2 $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

$$= \lim_{h \rightarrow 0} \frac{\left(\frac{3}{x+h-2} - \frac{3}{x-2} \right)}{h} = \frac{1}{h}$$

Step 3 $\lim_{h \rightarrow 0} \left(\frac{3}{x+h-2} - \frac{3}{x-2} \right) \frac{1}{h}$

Step 4 Find LCD (lowest common denominator)
LCD = $(x+h-2)(x-2)$

so: $\lim_{h \rightarrow 0} \left(\frac{3(x-2) - 3(x+h-2)}{(x+h-2)(x-2)} \right) \frac{1}{h}$

Step 5 Distribute and cancel out

so: $\lim_{h \rightarrow 0} \left(\frac{\cancel{3x} - \cancel{6} - \cancel{3x} - 3h + \cancel{6}}{(x+h-2)(x-2)} \right) \frac{1}{h}$

so: $\lim_{h \rightarrow 0} \left(\frac{-3h}{(x+h-2)(x-2)} \right) \frac{1}{h}$

When $h=0$; $\frac{-3}{(x-2)(x-2)} = \frac{-3}{(x-2)^2} = \text{derivative}$